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$$R = \text{radius} = \frac{AC}{2 \sin b} = \frac{1}{4} \sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(s-a)(s-b)(s-c)(s-d)}}$$

$$2R = D = \frac{1}{2} \sqrt{\frac{(39)(42)(38)}{(4)(6)(3)(5)}} = \frac{1}{2} \sqrt{172.9} = 6.57457225.$$

Also solved by *P. S. BERG, H. C. WHITAKER, J. R. BALDWIN, P. H. PHILBRICK, J. W. SCHEFFER.*

[Note.—The formula for the area of an inscriptable quadrilateral is

$$A = \sqrt{s(s-a)(s-b)(s-c)(s-d)}, \text{ where } s = \frac{1}{2}(a+b+c+d). \text{—ED.}]$$

7. Proposed by **WILLIAM HOOVER, A. M., Ph. D.,** Professor of Mathematics and Astronomy in the Ohio University, Athens, Ohio.

Through each point of the straight line  $x=my+h$  is drawn a chord of the parabola  $y^2=4ax$ , which is bisected in the point. Prove that this chord touches the parabola  $(y+2am)^2=8a(x-h)$ .

II. Solution by **L. E. PRATT, Tecumseh, Nebraska.**

Let  $y=Bx+C \dots (1)$  be a straight line cutting the given parabola. The co-ordinates of the middle point of the chord intercepted by the curve are

$$\left( \frac{2a-BC}{B^2}, \frac{2a}{B} \right).$$

Substituting these for  $x$  and  $y$  in the equations of the given straight line, we have

$$BC=2a-2amB-hB^2 \dots (2).$$

If  $(x_1, y_1)$  be any point of (1) we have  $y_1=Bx_1+C \dots (3)$ .

Eliminating  $C$  from (2) and (3) we obtain a quadratic in  $B$  which may be written

$$B = \frac{y_1+2am \pm \sqrt{(y_1+2am)^2-8a(x_1-h)}}{2(x_1-h)} \dots (4).$$

This result shows that two chords bisected by the straight line  $x=my+h$  may be drawn through the point  $x_1, y_1$ ; that when the roots of  $B$ , the angular coefficient, are equal the two chords coincide in one; that this takes place when the radical in (4) is equal to zero. But when the radical equals zero the point  $x_1, y_1$ , is a point of the parabola

$$(y+2am)^2=8a(x-h)$$

and the straight line (1) is tangent to it.

This problem was solved in a very excellent manner by *Professors HUME, SCHEFFER, and ZERR.*

9. Proposed by **J. C. GREGG, Superintendent of Schools, Brazil, Indiana.**

Two circles intersect in  $A$  and  $B$ . Through  $A$  two lines  $CAE$  and  $DAF$  are drawn, each passing through a centre and terminated by the circumference. Show that  $CA \times AE = DA \times AF$ . [*Euclid.*]

Solution by **Miss GRACE H. GRIDLEY, Student in Kidder Institute, Kidder, Missouri.**

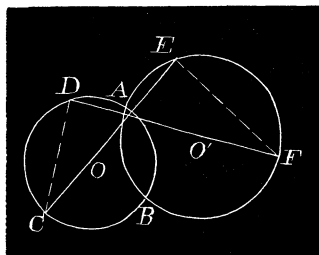
Let the straight lines  $CAE$  and  $DAF$  pass through the point of intersection  $A$  of the two circles, and through the centers  $O$  and  $O'$ , respectively.

Connect by straight lines the points  $D$  and  $C$ , and  $E$  and  $F$ . Then the triangles  $CDA$  and  $AED$  are right triangles, being inscribed in a semi-circle. The two triangles are also similar, having the acute angle  $EAF$  of the one equal to the acute angle  $DAC$  of the other.

$\therefore CA : AF :: AD : AE$ , the homologous sides of similar triangles being proportional.

$\therefore CA \times AE = AF \times AD$ . Q. E. D.

*Remark.*—When the angle  $CAF$  is a right angle  $AE$  and  $AD$  each equal zero. When the angle  $CAF$  is less than a right angle, the point  $E$  will fall on the semi-circumference  $ABF$  and the point  $D$  will fall on the diameter  $AF$  and  $CA \times AE = AF \times AD$  as before.



Also solved by J. F. W. SCHEFFER, JOSIAH H. DRUMMOND, ROBERT J. ALEY, G. B. M. ZERR, J. A. CALDERHEAD, P. S. BERG, and P. H. PHILBRICK.

19. Proposed by ERIC DOOLITTLE, Instructor in Mathematics, State University of Iowa, Iowa City.

If  $MN$  be any plane, and  $A$  and  $B$  any points without the plane, to find a point  $P$ , in the plane, such that  $AP + PB$  shall be a minimum.

Solution by THADDEUS MERRIMAN, South Bethlehem, Pennsylvania.

First, suppose the points to be on opposite sides of the plane; the point where the straight line joining the given points pierces the plane is the required point  $P$ ; since a straight line is the shortest distance between two points.

Second, suppose the points to be on the same side of the plane; let  $AB$  be the straight line joining them,  $CD$  the projection of  $AB$  on the plane  $MN$ , and  $AC$  the perpendicular which projects  $A$  on the plane. Now, produce  $AC$  to  $E$  making  $AC = CE$ , and join  $E$  and  $B$ ; the point  $P$  where  $EB$  cuts  $CD$  is the required point. For, join  $A$  and  $P$ , and let  $Q$  be any other point in the plane; join  $A$  and  $Q$ , and  $B$  and  $Q$ , also  $Q$  and  $E$ . Now, since  $AC = CE$  by construction,  $AP = PE$  and  $AQ = QE$ ; consequently  $BQ + QE = BQ + QA$ , and therefore, since  $BE < BQ + QE$ , we have  $AP + PB < AQ + QB$ , or  $AP + PB$  is a minimum.

[This demonstration is by Thaddeus Merriman, the 17 year old son of Professor Mansfield Merriman.—Ed.]

Also solved by J. H. BEACH, G. B. M. ZERR, LEONARD E. DICKSON, F. A. SWANGER, H. C. WHITAKER, P. H. PHILBRICK, A. H. BELL and J. F. W. SCHEFFER.

## PROBLEMS.

31. Proposed by Professor G. I. Hopkins, Manchester, New Hampshire.

A field is bounded as follows: N.  $14^\circ$  W. 15.2 chains; N.  $70^\circ 30'$  E. 20.43 chains; S.  $6^\circ$  E. 22.79 chains; N.  $86^\circ 30'$  W. 18 chains. A spring within it bears from the second corner S.  $75^\circ$  E. 7.9 chains. It is required to cut off 10 acres from the west side of the field by a straight fence through the spring. How far will it be from the first corner to the point at which the division fence meets the fourth side?